

**International Conference
on
Probabilistic Political Economy
"Laws of Chaos" in the 21st Century.**

July 14-17, 2008 at Kingston Hill, UK

On the production
of labour value and use value
in capitalist and pre-capitalist worlds

Peter Karl Fleissner, Vienna, Austria

<http://transform.or.at>

$$f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0 \quad f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0$$

outline

1. setting the stage
2. first empirical results for Austria 2003
3. the role of services and productivity measures
4. geometric interpretation of prices, values and volumes
5. transformation problem revisited
6. Iterative solution: empirical results
7. generalized transformation problem: moving the tip of the value vector in a hyperplane

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Setting the stage: Basic terms in Marxian Political Economics

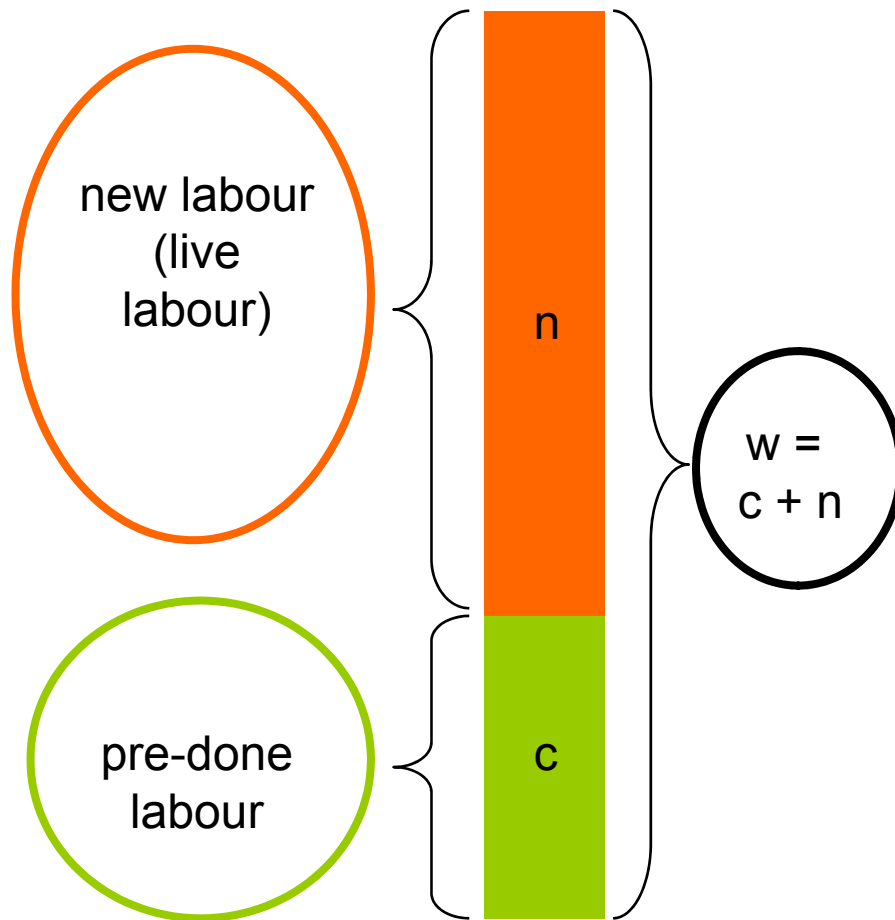
- commodity
- value in use
- value in exchange

- (labour)value
- constant capital
- variable capital
- surplus value

- rate of surplus value/rate of exploitation
- organic composition of capital
- rate of profit

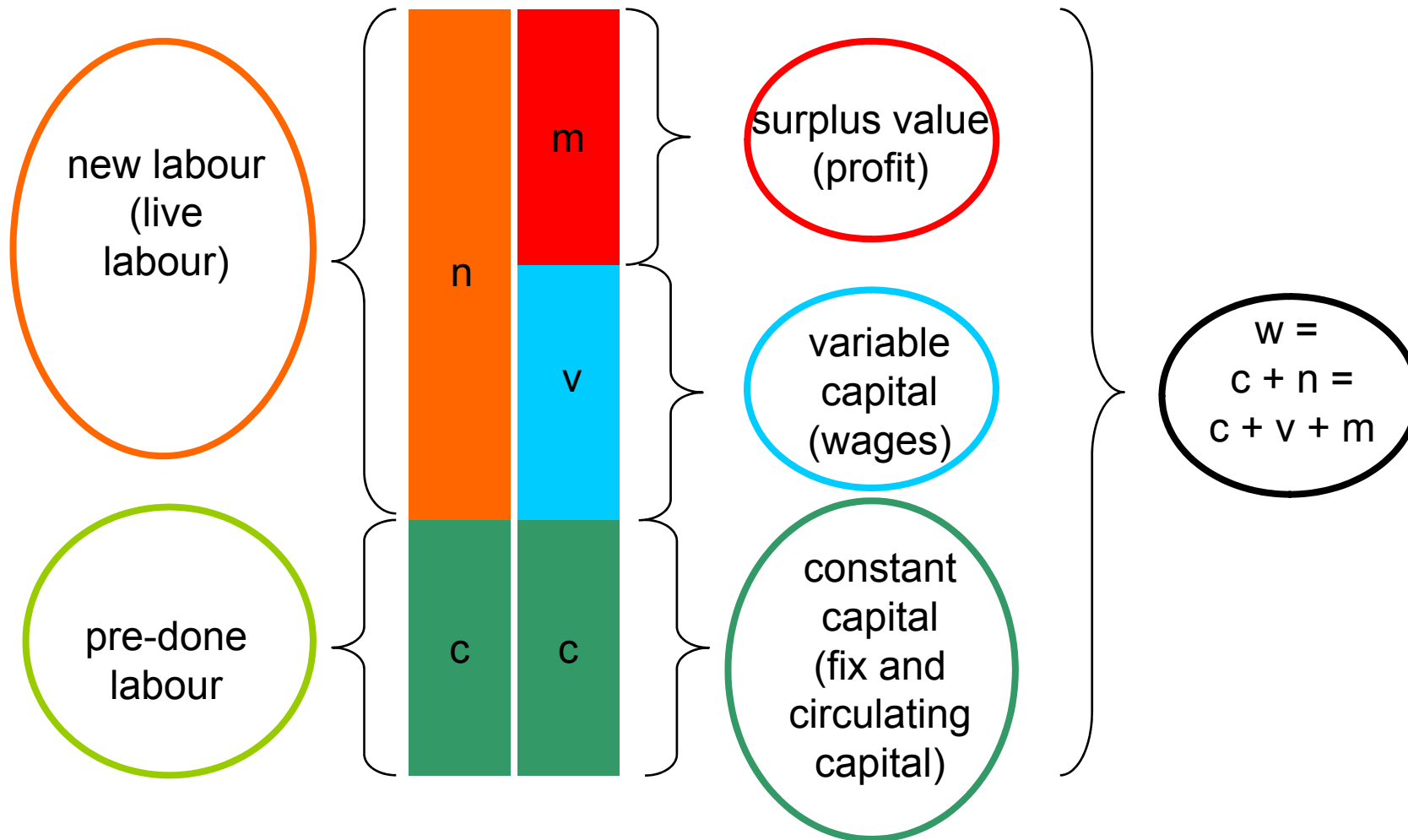
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the composition of labour value w



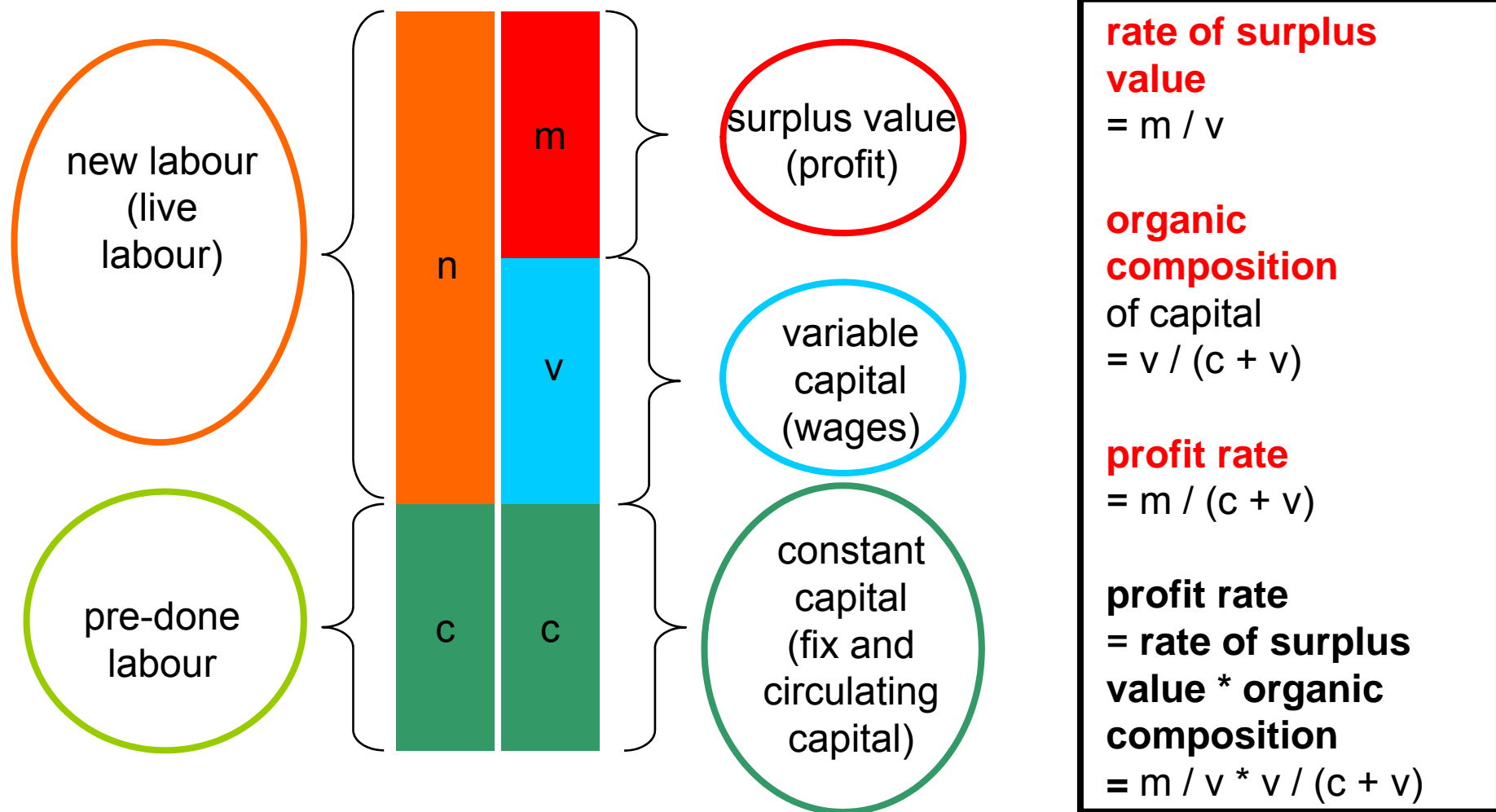
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labour value w and its composition



$$f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0 \quad f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0$$

three essential Marxian indicators



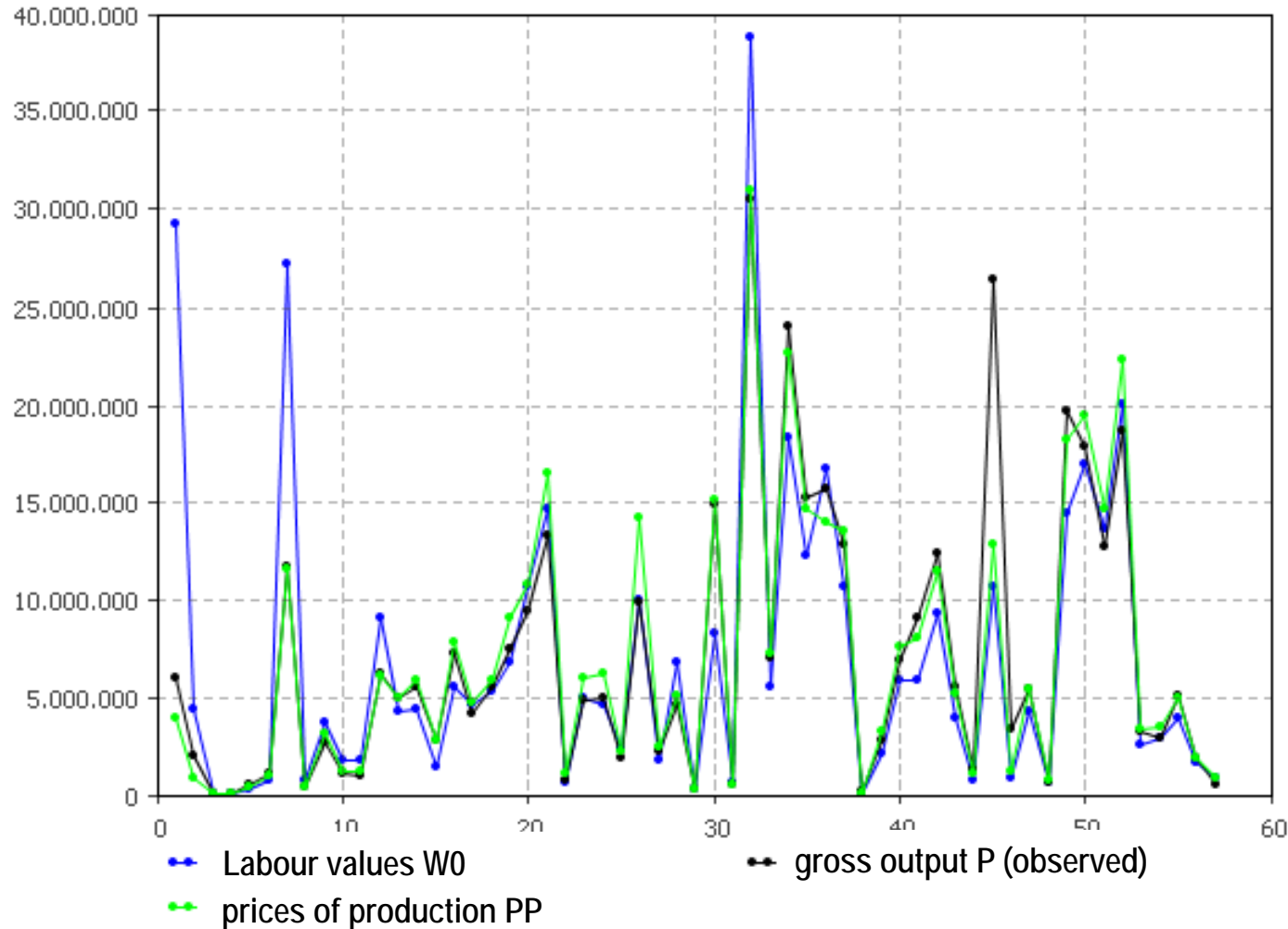
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Empirical results:
 Gross-output (P), labour values (W0) and prices of production (PP)
 Austria 2003: 57 industries (Mio EUR)



Correlation coefficient with	r
classical labour values w	0.883
Prices of production pp (Bortkiewicz)	0.952

$$f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0 \quad f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0$$

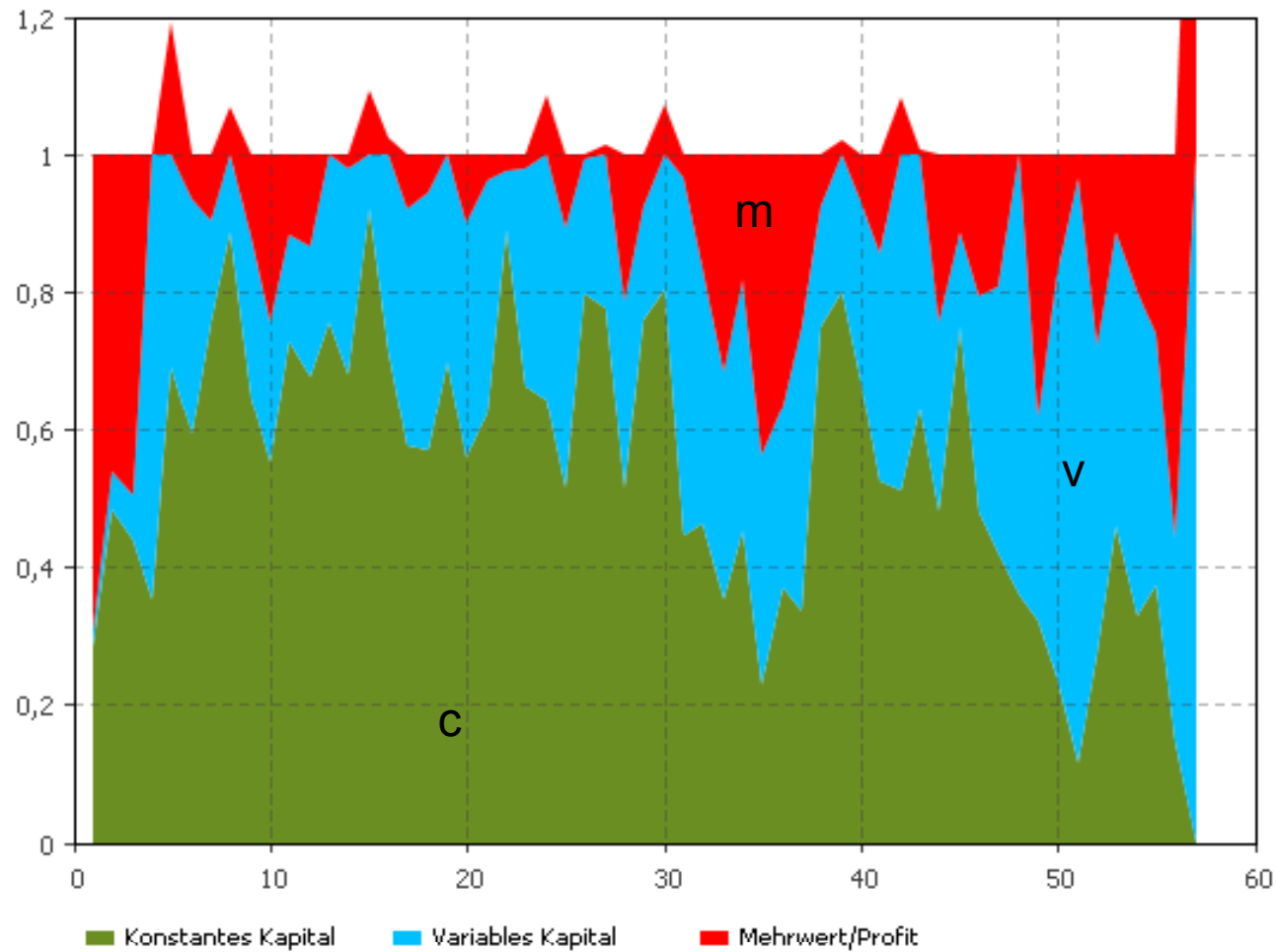
Nr	Industry
1	Agriculture, hunting
2	Forestry, logging
3	Fishing, fish farms
4	Mining of coal and lignite
5	Extract. o. crude petrol. a. nat. gas, min. o. metal ores
6	Other mining and quarrying
7	Manufacture of food products and beverages
8	Manufacture of tobacco products
9	Manufacture of textiles
10	Manufacture of wearing apparel
11	Manufacture of leather, leather products, footwear
12	Manufacture of wood and of products of wood
13	Manufacture of paper and paper products
14	Publishing, printing and reproduction
15	Manufacture of coke, refined petroleum products
16	Manufacture of chemicals and chemical products
17	Manufacture of rubber and plastic products
18	Manufacture of other non-metallic mineral products
19	Manufacture of basic metals
20	Manufacture of fabricated metal products
21	Manufacture of machinery and equipment n.e.c.
22	Manufacture of office machinery and computers
23	Manufacture of electrical machinery and apparatus n.e.c.
24	Manufacture of radio, television equipment
25	Manuf. of medical, precision, optical instruments, clocks
26	Manufacture of motor vehicles and trailers
27	Manufacture of other transport equipment
28	Manufacture of furniture; manufacturing n.e.c.
29	Recycling
30	Electricity, gas, steam and hot water supply
31	Collection, purification and distribution of water
32	Construction
33	Sale and repair of motor vehicles; automotive fuel
34	Wholesale and commission trade
35	Retail trade, repair of household goods
36	Hotels and restaurants
37	Land transport; transport via pipelines
38	Water transport
39	Air transport
40	Supporting a. auxiliary transport activities; travel agencies
41	Post and tele-communications
42	Financial intermediation, except insur.
43	Insurance and pension funding, except social security
44	Activities auxiliary to financial intermediation
45	Real estate activities
46	Renting of machinery and equipment without operator
47	Computer and related activities
48	Research and development
49	Other business activities
50	Public administration; compulsory social security
51	Education
52	Health and social work
53	Sewage and refuse disposal, sanitation and similar act.
54	Activities of membership organizations n.e.c.

Structure of „classical“ labour values

all industries are value producers

c - constant capital, v - variable capital, m - surplus value

Austria 2003: 57 industries (percent)

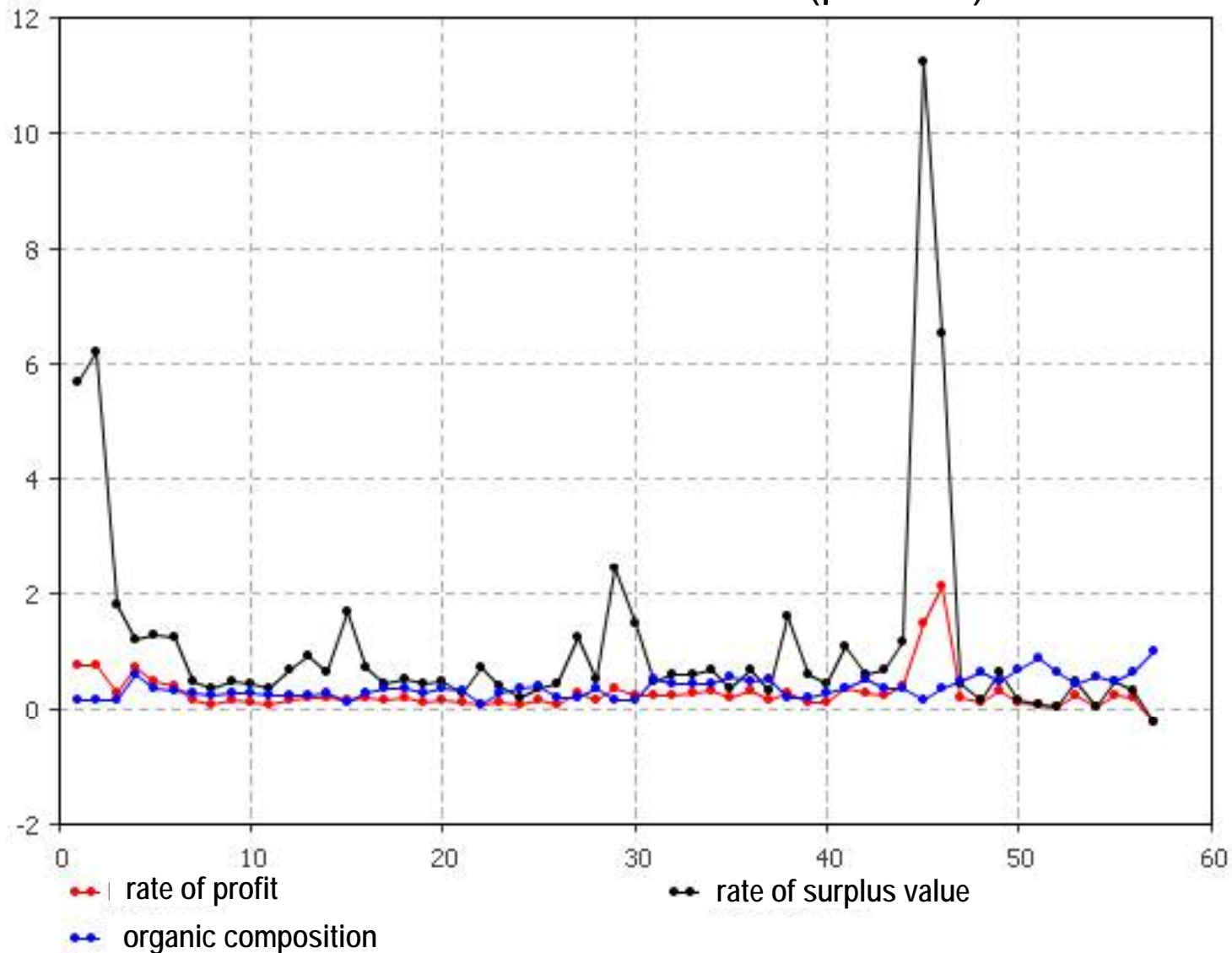


$$f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0$$

Marxian indicators (observed data)

rate of surplus value, organic composition of capital, rate of profit

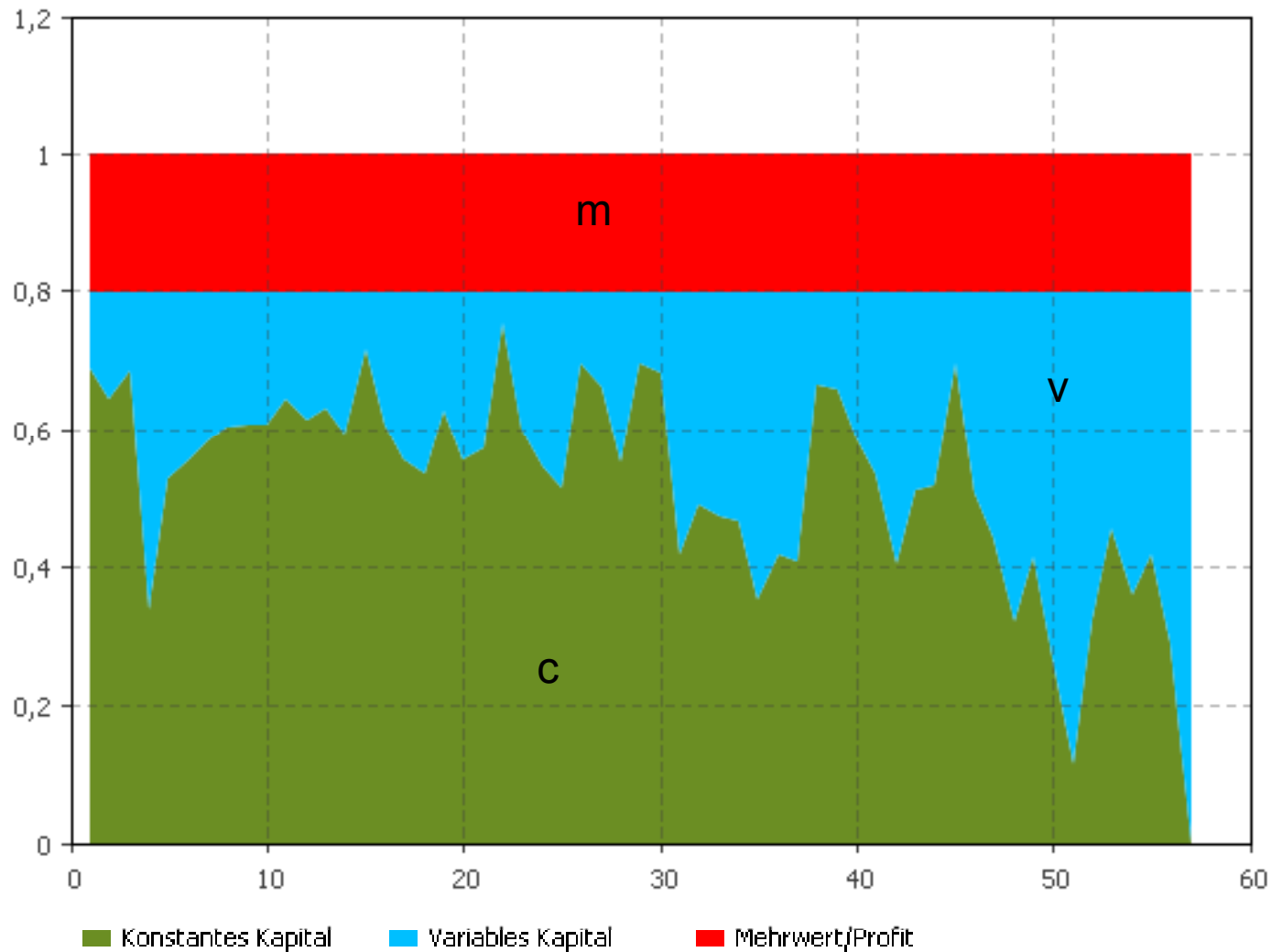
Austria 2003: 57 industries (percent)



von Bortkiewicz: Prices of production

c - constant capital, v - variable capital, m - surplus value

Austria 2003: 57 industries (percent)



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Two sources of inspiration

- Productive and unproductive labour (Adam Smith)
- Material Product System (MPS) vs System of National Accounts (SNA – Richard Stone)

$$f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0 \quad f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0$$

The labour of some of the most respectable orders in the society is, like that of menial servants, unproductive of any value, and does not fix or realize itself in any permanent subject; or vendible commodity, which endures after that labour is past, and for which an equal quantity of labour could afterwards be procured.....In the same class must be ranked, some both of the gravest and most important, and some of the most frivolous professions: churchmen, lawyers, physicians, men of letters of all kinds; players, buffoons, musicians, opera-singers, opera-dancers, &c. ...Like the declamation of the actor, the harangue of the orator, or the tune of the musician, the work of all of them perishes in the very instant of its production.

Smith, Adam, [An Inquiry into the Nature and Causes of the Wealth of Nations](#),
Book II, Chapter III, Of the Accumulation of Capital, or of Productive and
Unproductive Labour, <http://www.econlib.org/LIBRARY/Smith/smWN.html>.

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treating services as value consuming

- there is an essential difference between *goods* (= *material products*) and *services*
- **services** do not contribute neither to surplus product, nor to surplus value, they as such cannot be resold nor accumulated nor invested, because they are consumed when they are produced
- In „Das Kapital“, Vol I, Marx dealt only with *material products* where according to his **labour theory of value (LTV)** the **principle of equivalent exchange** holds.
- **principle of equivalent exchange** : goods are exchanged according to their content of social necessary labour
- **if service sectors are allowed to make profits** (as it is the case under capitalism), **the principle of equivalent exchange is violated and LTV is no longer valid**

$$f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0 \quad f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0$$

How to determine labour values that obey the principle of equivalent exchange?

- A... partitioned matrix of technical coefficients
- C... partitioned matrix of unit consumption
- R... partitioned matrix of unit reproduction
- n... partitioned row vector of unit live labour = $\{ n_1, n_2 \}$
- w... partitioned row vector of labour values = $\{ w_1, w_2 \}$
- I.... Identity matrix

$$A = \begin{Bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{Bmatrix}, \quad C = \begin{Bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{Bmatrix}, \quad R = A + C$$

w: „classical“ labour values: all industries are value producer
 $w = n (I - A)^{-1}$

w*: value production in material sectors only

$$w^* = \{ n_1(I - A_{11})^{-1}, n_1(I - A_{11})^{-1} (A_{12} + C_{12}) [I - (A_{22} + C_{22})]^{-1} \}$$

$$f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0 \quad f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0$$

Construction of the consumption matrix C in analogy to A

- Consumption matrix C (on unit-level)

$$C = c v \text{diag}(x) / (vx) \text{diag}(x)^{-1} = c v / (vx)$$

c... column vector of consumption

v... row vector of unit wages

x... column vector of output

p... row vector of unit prices

$$pC = v$$

$$Cx = c$$

$$f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0 \quad f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0$$

Construction of the surplus matrix S in analogy to A

- Surplus Matrix S (on unit-level)

$$S = s \, m \, \text{diag}(x) / (m \, x) \, \text{diag}(x)^{-1} = s \, m / (m \, x)$$

s... column vector of surplus product

m... row vector of unit surplus value (or unit profit)

x... column vector of output

p... row vector of unit prices

$$pS = m$$

$$Sx = s$$

$$f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0 \quad f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0$$

Services

in a simplified Leontief economy

Leontief economy:

$$\begin{array}{ll} \text{primal:} & (A + C + S)x = x \\ \text{dual:} & p(A + C + S) = p \end{array}$$

S... partitioned matrix of unit surplus product

$$S = \begin{Bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{Bmatrix} \quad \text{surplus product } s = (I - A - C)x$$

If services are present, $S_{21} = 0$ and $S_{22} = 0$,
because services do not contribute to surplus product

Sub-matrix S_{12} is crucial: If principle of equivalent exchange holds,
 $S_{12} = 0$.

- Material producers can fully invest,
- services cannot invest at all

$$f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \quad \text{for } x > 0 \quad f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \quad \text{for } x > 0$$

Effects on the service sector

- Growing \mathbf{S}_{12} \rightarrow higher service prices

$$p^*_2 = p_2 + p_1 \mathbf{S}_{12} (\mathbf{E} - \mathbf{R}_{22})^{-1}; \quad x^*_2 = x_2$$

- Reallocation of $\mathbf{C}_{11}x_1$, consumption in sector 1 towards services $\rightarrow x_2$ increases

$$x^*_2 = x_2 + \mathbf{C}_{11} x_1 / (\mathbf{A}_{12} + \mathbf{C}_{12}); \quad p^*_2 = p_2$$

- Multiple accounting under SNA (seen from MPS)

Double accounting, if diagonal matrices=0

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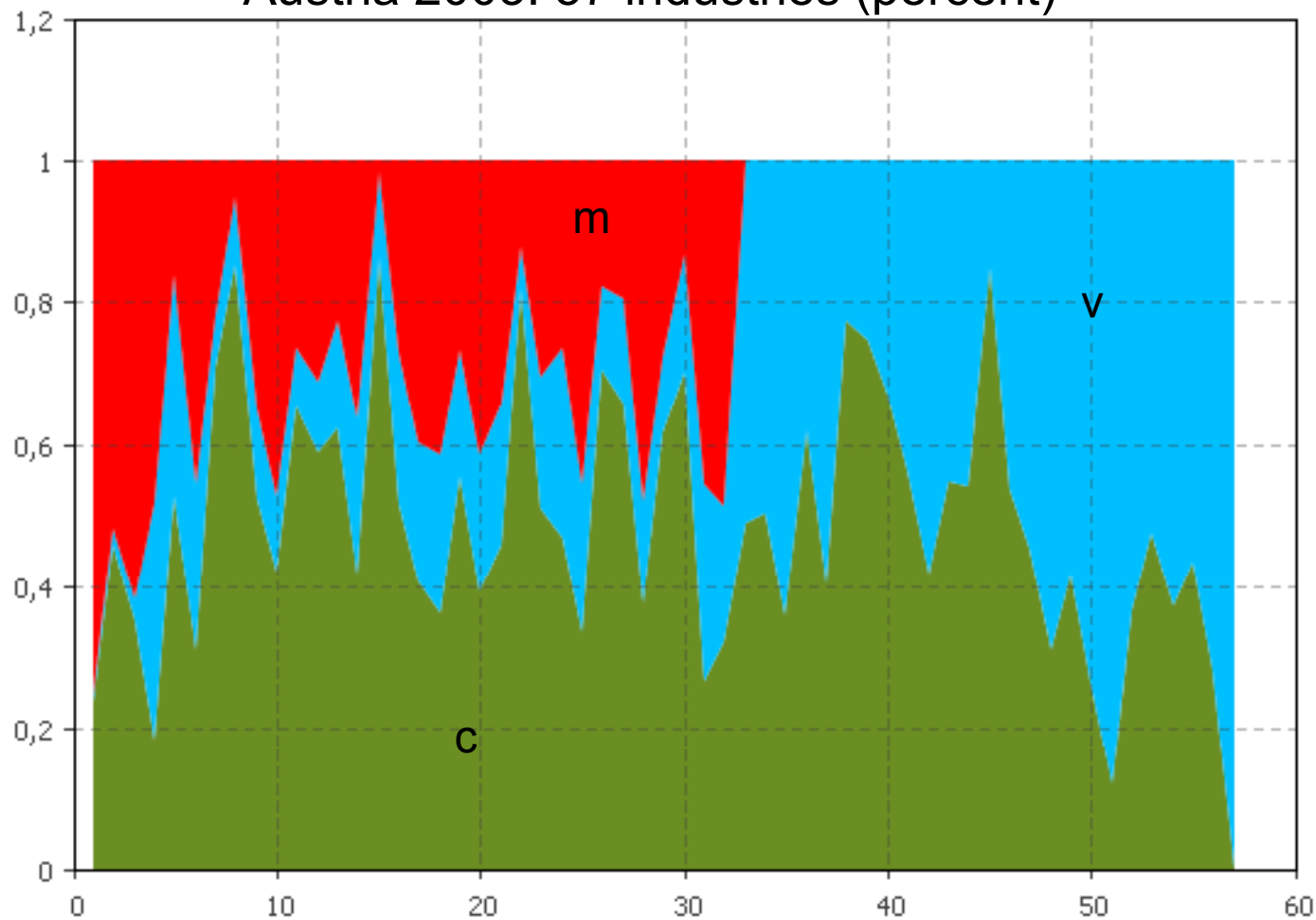
Three measures of productivity

- *Productivity(1)*, productivity of use value, number of use-values per working hour (independent of relations of production)
- *Productivity(2)*, productivity of labour value, 0 or 1 (productive vs. unproductive labour: Adam Smith). Important to characterize the difference between goods and services
- *Productivity(3)*, productivity of profit, measured by profit over live labour in working hours. Productivity measure of capitalism, proposed by Marx

$$f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0 \quad f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0$$

Structure of labour values

no surplus for services, **variable** exploitation rates
c - constant capital, v - variable capital, m - surplus value
Austria 2003: 57 industries (percent)



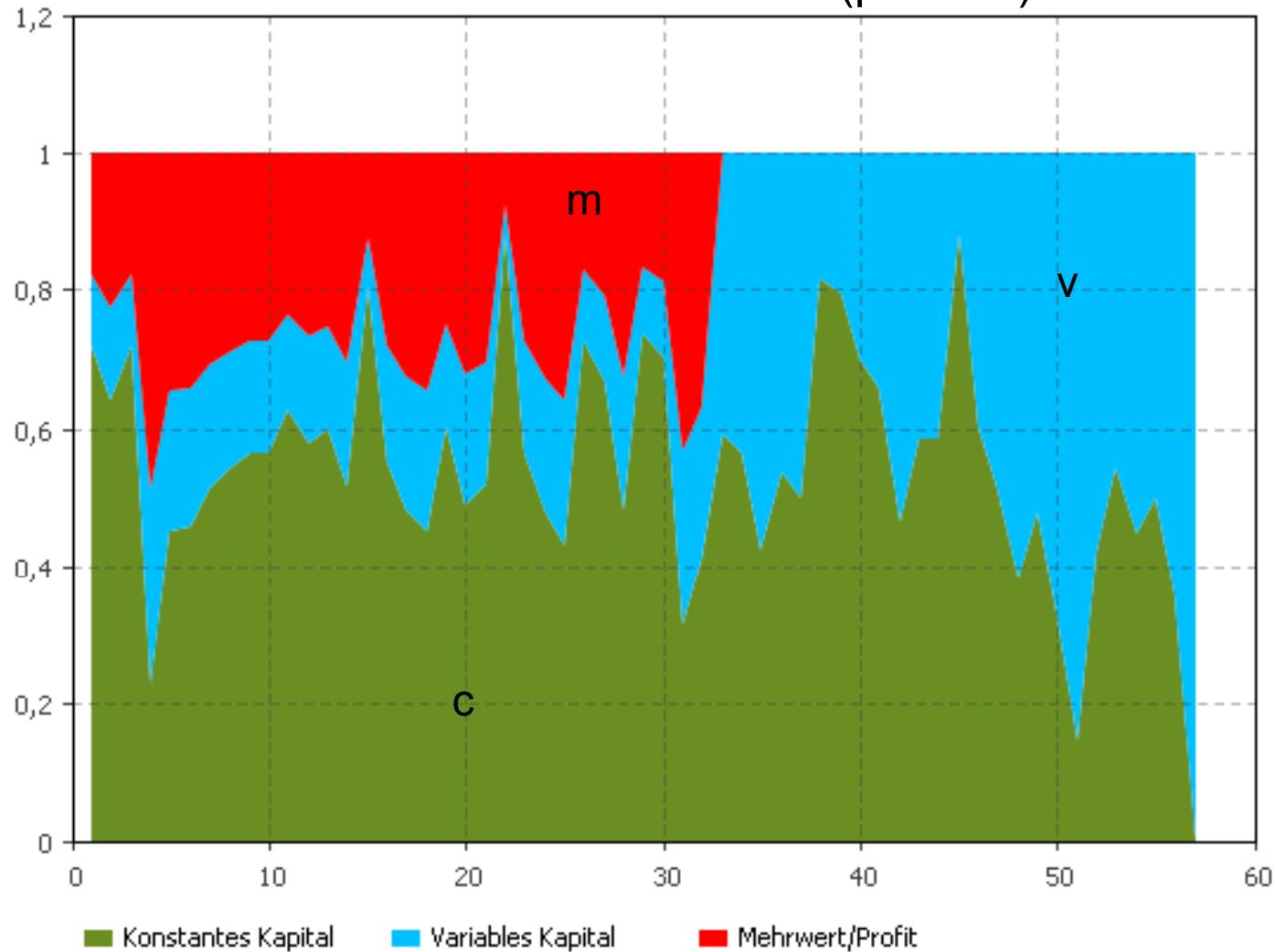
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Structure of labour values

no surplus for services, **equal** exploitation rates

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Marxian indicators

exploitation rates (**equal**), organic composition of capital, rate of profit

Austria 2003: 57 industries (in percent)



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geometric interpretation of prices, values and volumes

More variables

r... rate of profit

g... rate of growth

Marx' notation

$w = \text{"c"} + \text{"v"} + \text{"m"}$

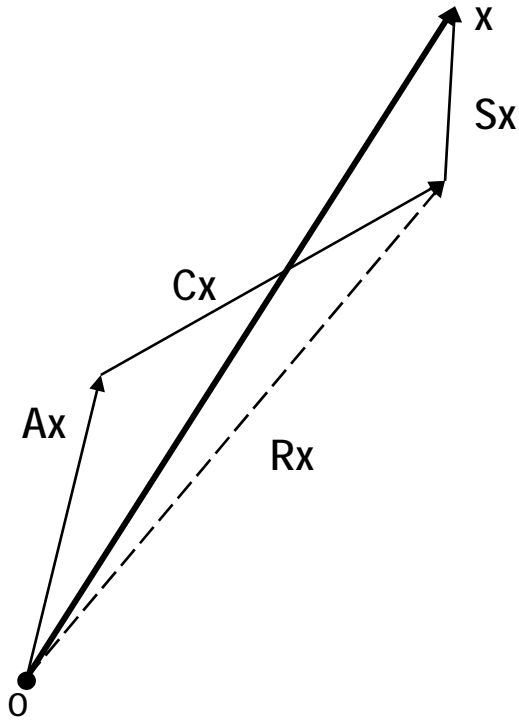
	row vector on unit level	turnover level
Constant circulating capital	"c" = wA	= wA diag(x)
Variable capital	"v" = wC	= wC diag(x)
Surplus value	"m" = Ws	= wS diag(x)

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Dual decomposition of

output x (left)

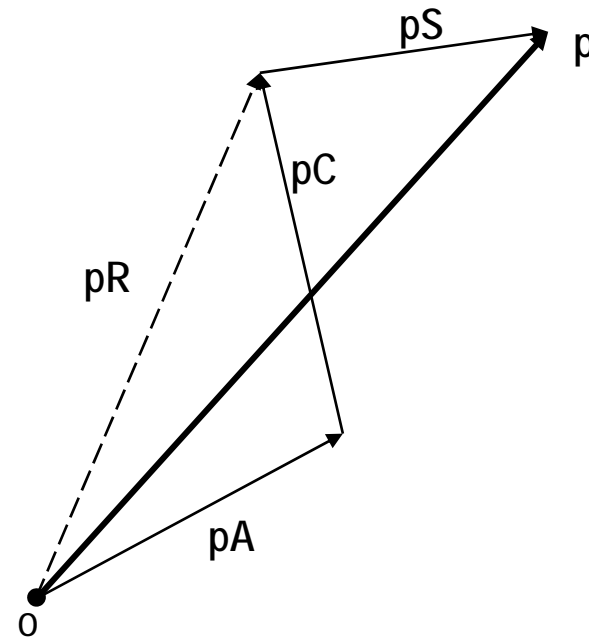
$$Ax + Cx + Sx = x = \\ = Rx + Sx = x$$



and

unit prices p (right)

$$pA + pC + pS = p = \\ = pR + pS = p$$



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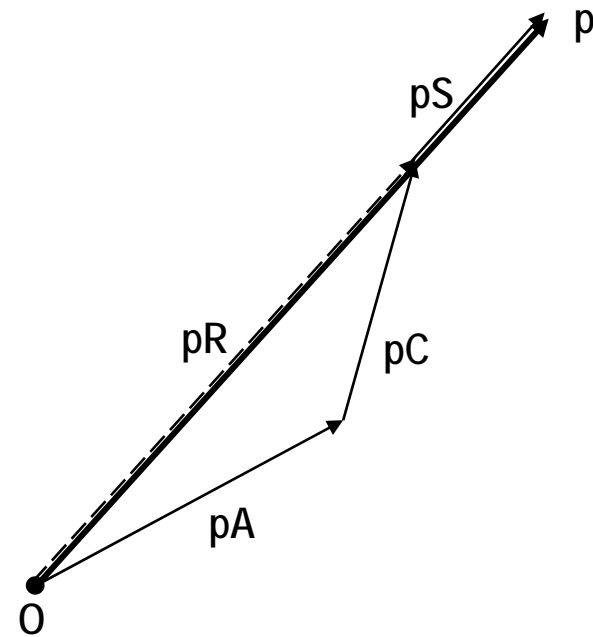
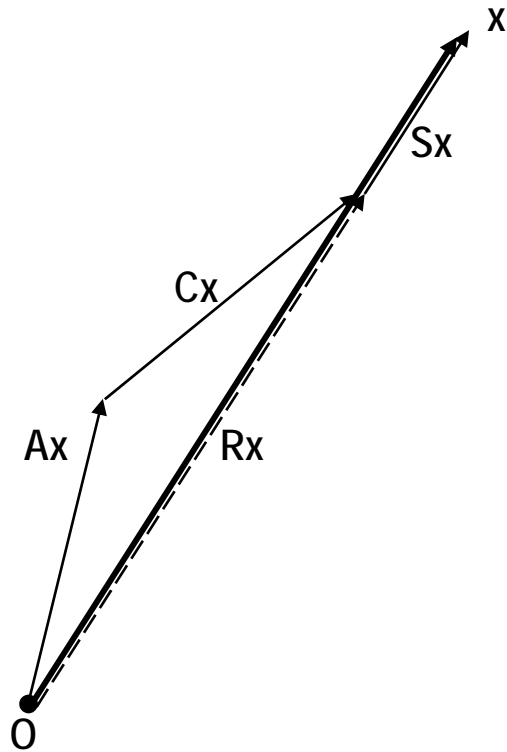
Dual decomposition of
output x (left) for equilibrium growth
and for unit prices of production p (right)

$$x = Rx (1 + g) =$$

$$= Rx + g Rx = Rx + Sx$$

$$p = pR (1 + r) =$$

$$pR + r pR = pR + pS$$



$$f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0$$

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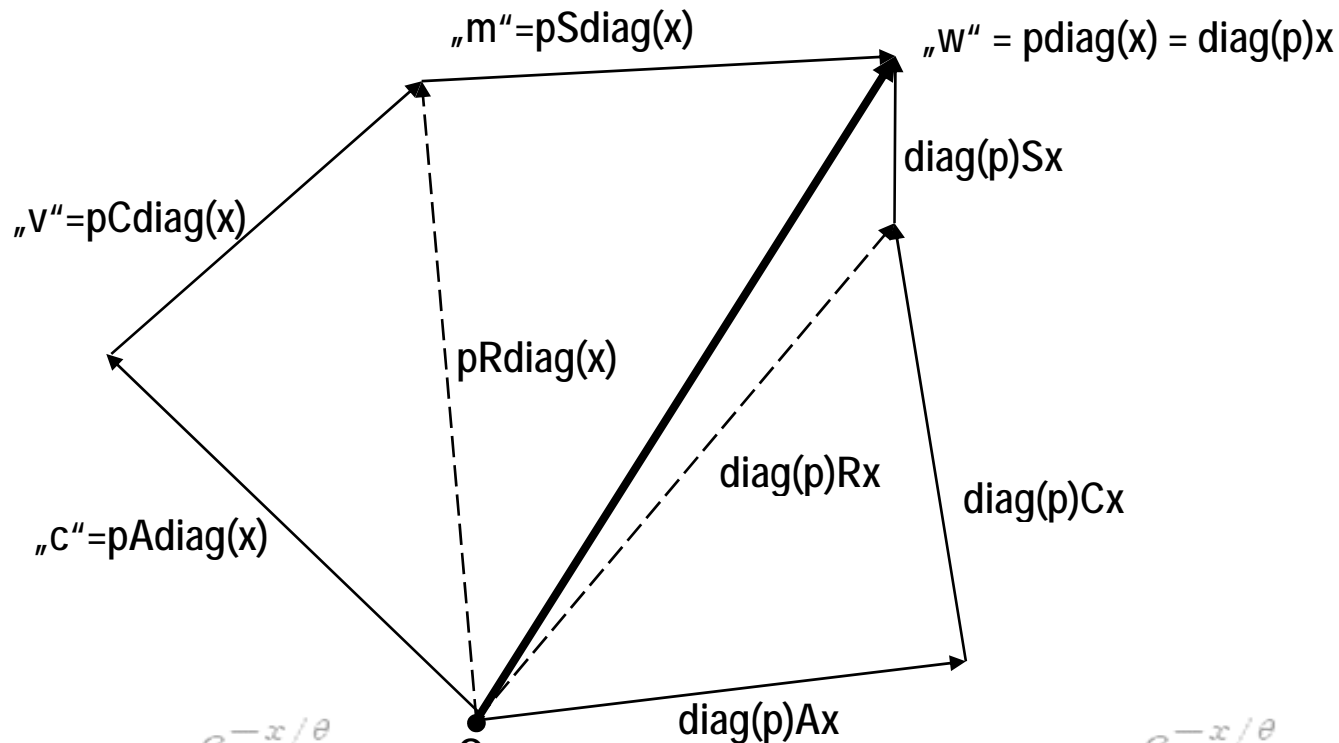
Dual decomposition of turnover w

“w” = pdiag(x) or “w” = diag(p)x

“w” = “c” + “v” + “m”, r = g

$w = pA\text{diag}(x) + pC\text{diag}(x) + pS\text{diag}(x)$

$w' = \text{diag}(p)Ax + \text{diag}(p)Cx + \text{diag}(p)Sx$



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transformation of labor values into prices (transformation problem)

Marx' solution

$$pp(0) = w \text{ or } w^*$$

$$pp(1) = pp(0) R [1 + r]$$

$$1 + r = [pp(0) x] / [pp(0) R x]$$

Difficulty: input prices \neq output prices

von Bortkiewicz solution

two identical solutions with different
philosophical implications

a) Eigenvector solution: $pp \dots$ left-hand Eigenvector of R
 $pp R (1 + r) = pp$, largest eigenvalue of R : $\lambda = 1/(1+r)$

b) iterative solution: $i \rightarrow \infty$ Marx' solution: $i = 1$

$$pp = pp(\infty); pp(0) = w \text{ or } w^*$$

$$pp(i) = pp(i-1) R [1 + r(i-1)], 1 + r(i) = [pp(i) x] / [pp(i) R x]$$

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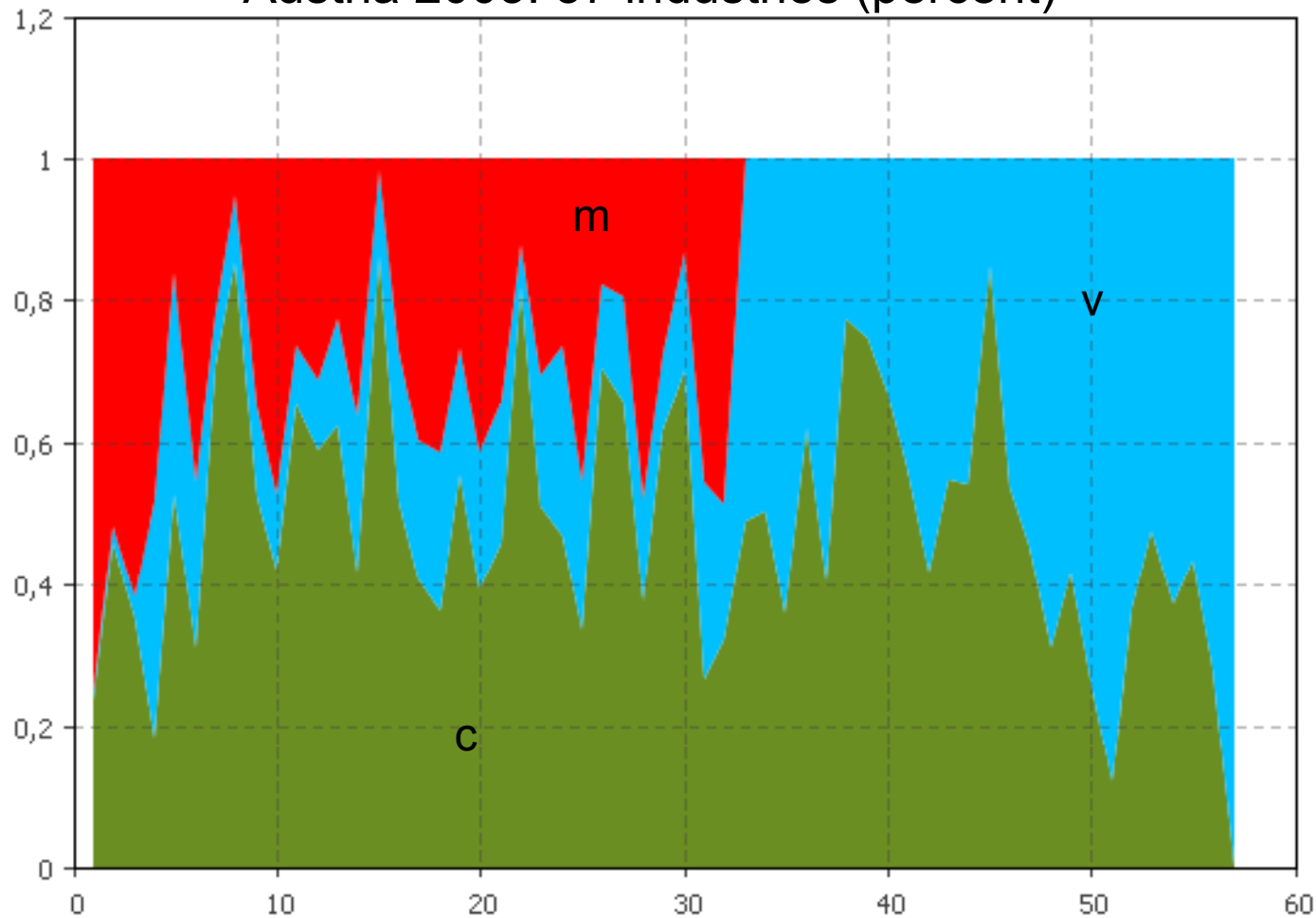
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c - constant capital, v - variable capital, m - surplus value
Austria 2003: 57 industries (percent)

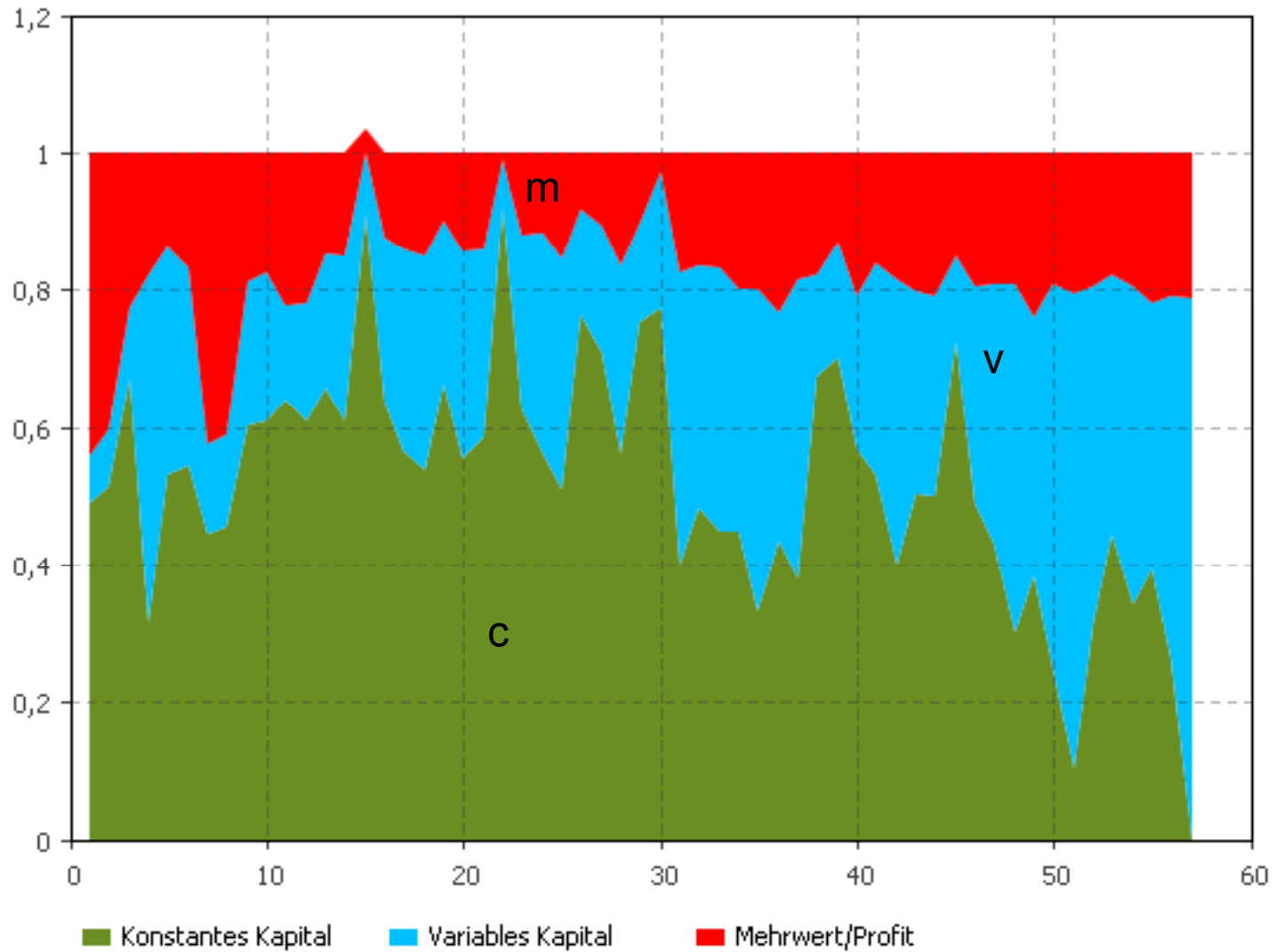


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Marx' solution: Prices of production

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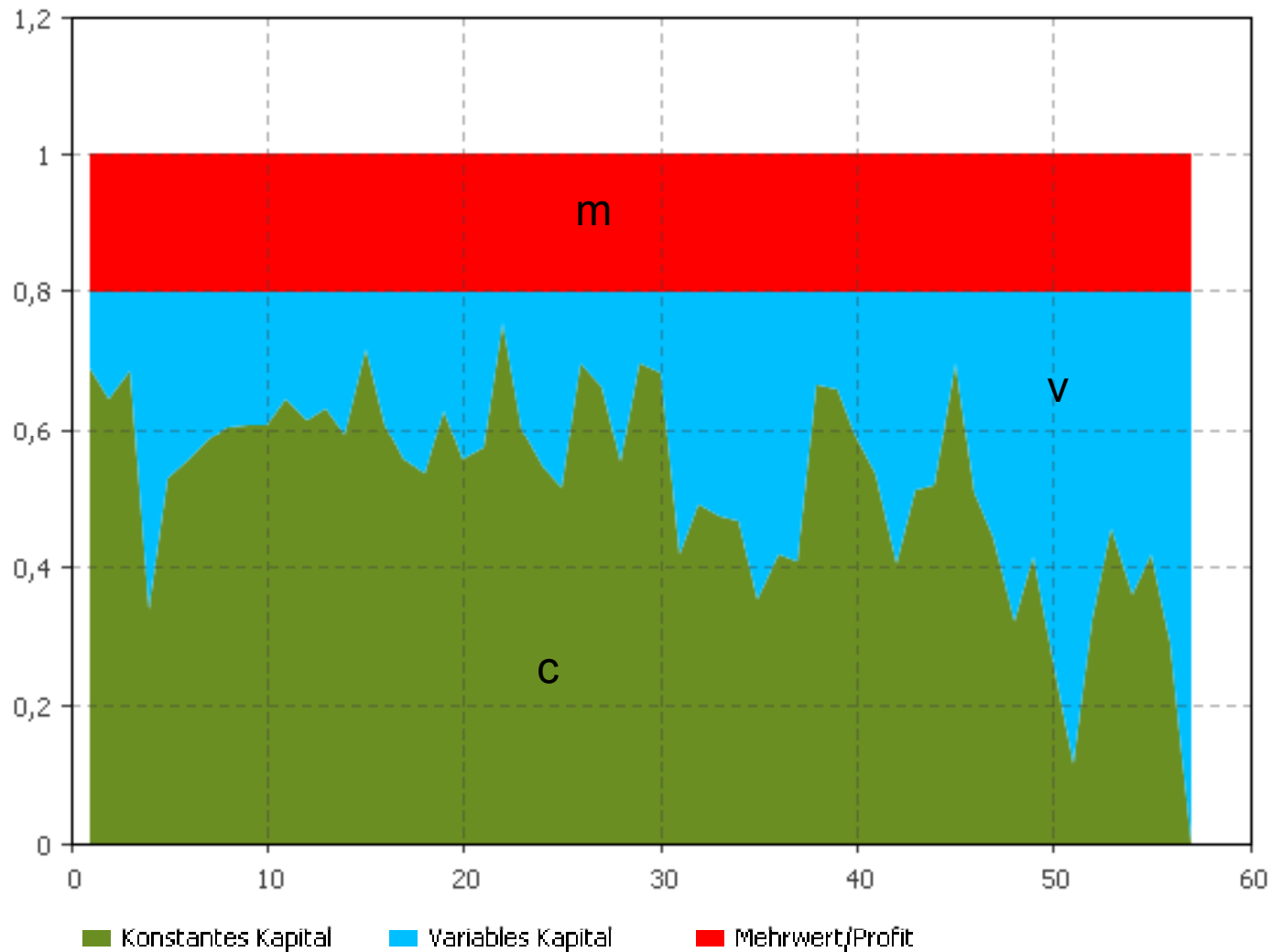
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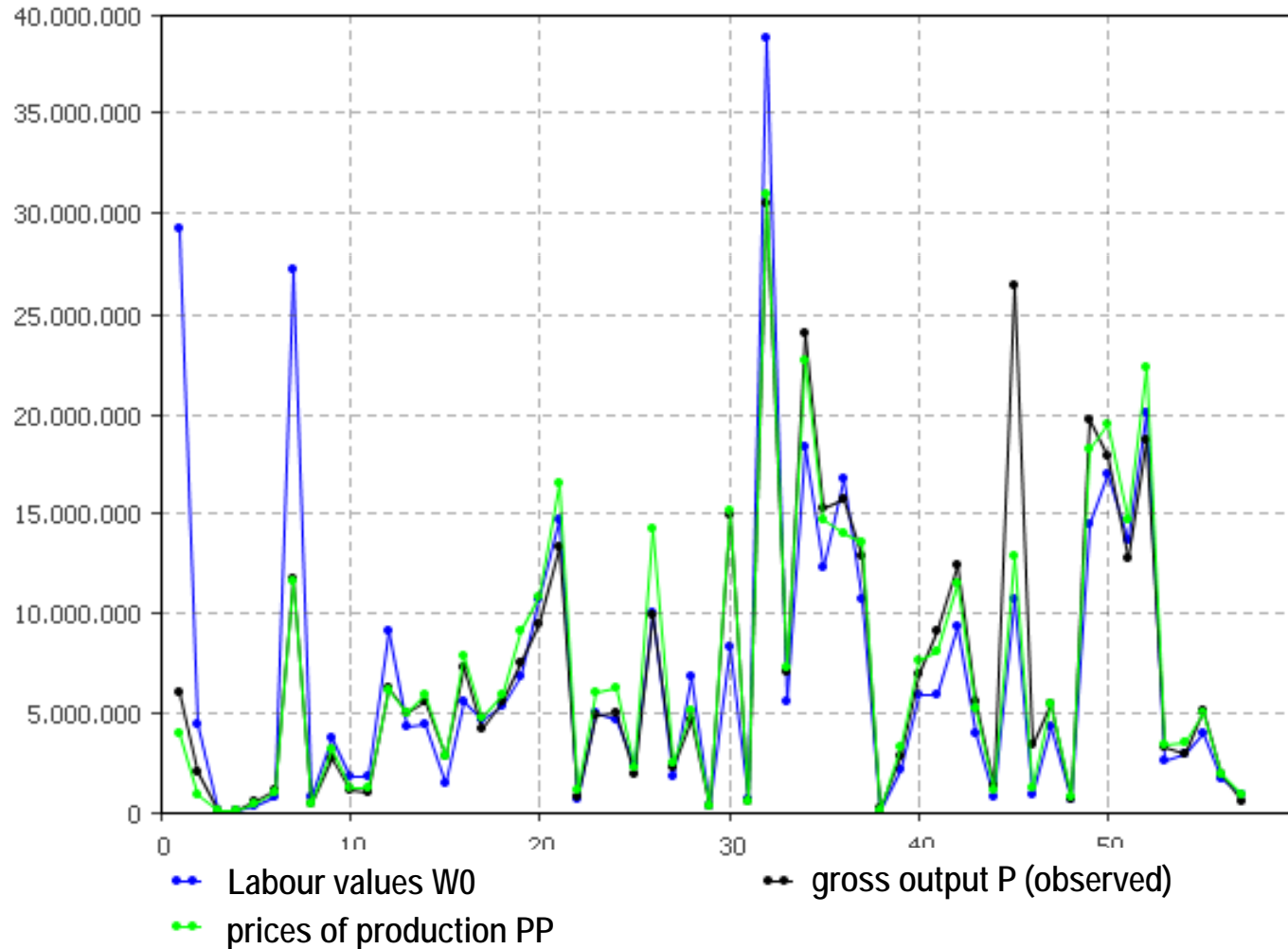
von Bortkiewicz: Prices of production

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Empirical results II:
Gross-output (P), labour values (W0) and prices of production (PP)
Austria 2003: 57 industries (Mio EUR)



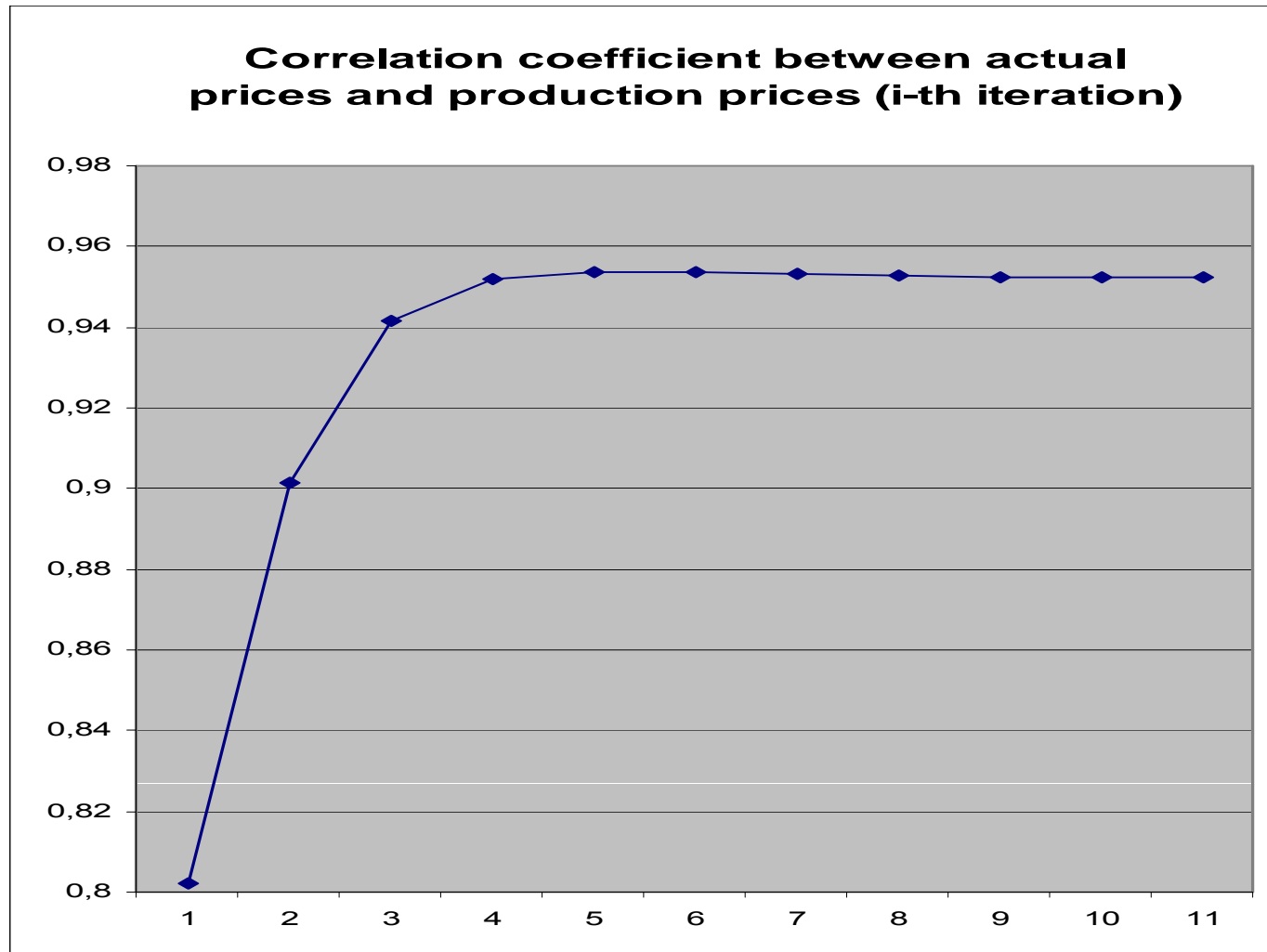
Correlation coefficient of p with	r
w - all sectors	0.883
w* – only mat. production	0.802
pp Marx first iteration	0.901
pp Marx 5th iteration	0,954
pp Bortkiewicz	0.952

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Transformation problem iterative solution



Iteration	Correlation
1	0,80200000
2	0,90131617
3	0,94169690
4	0,95211631
*	5 0,95373425
6	0,95349443
7	0,95306224
8	0,95273999
9	0,95253944
10	0,95242360
11	0,95235923



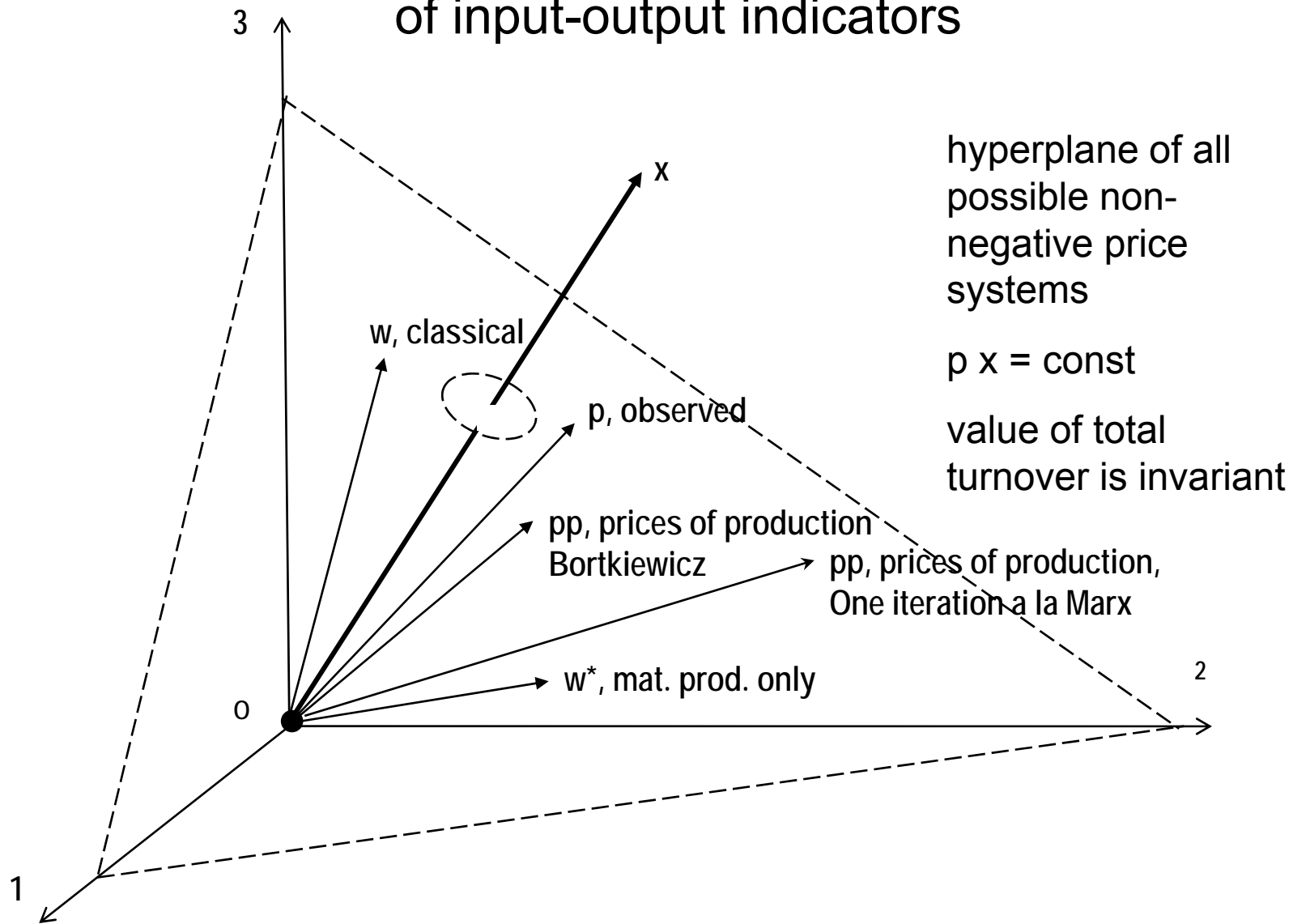
$$f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0 \quad f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0$$

outline

1. setting the stage
2. first empirical results for Austria 2003
3. the role of services and productivity measures
4. geometric interpretation of prices, values and volumes
5. transformation problem revisited
6. Iterative solution: empirical results
7. **generalized transformation problem:
moving the tip of the value vector in a
hyperplane**

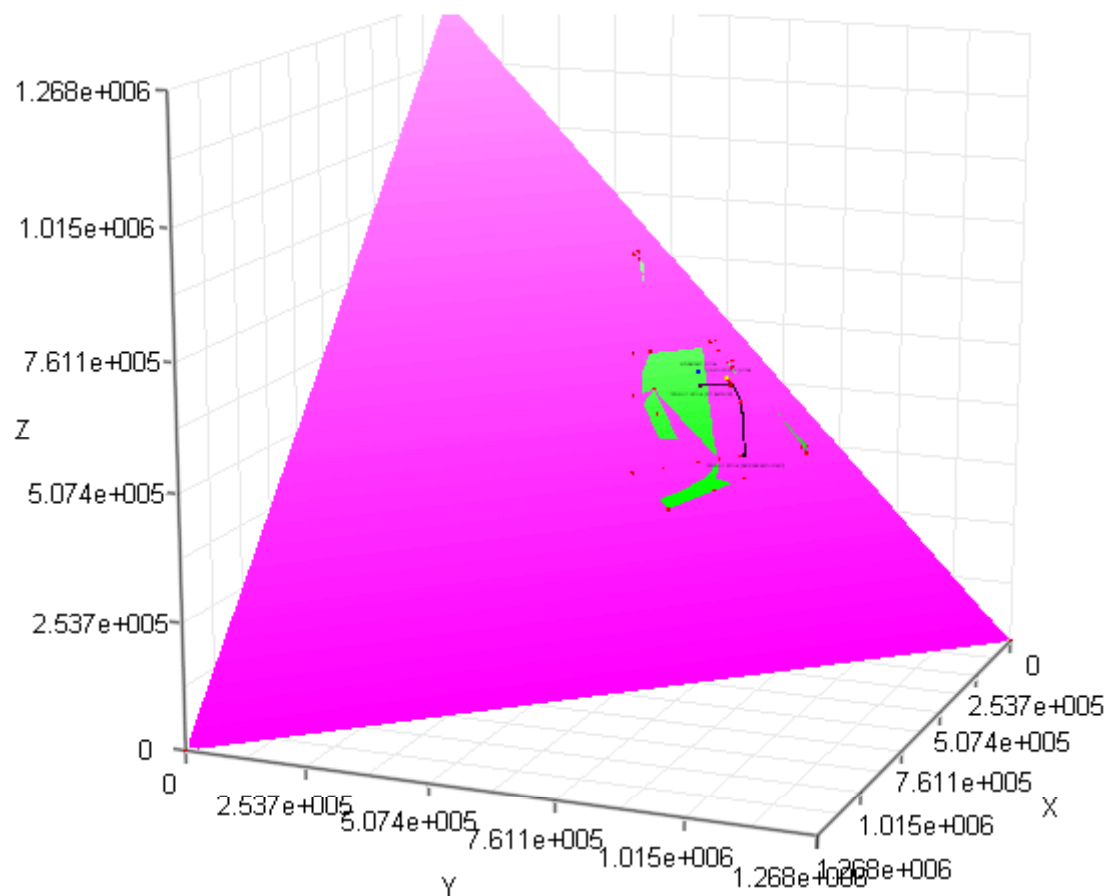
$$f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0 \quad f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0$$

Geometric interpretation of input-output indicators



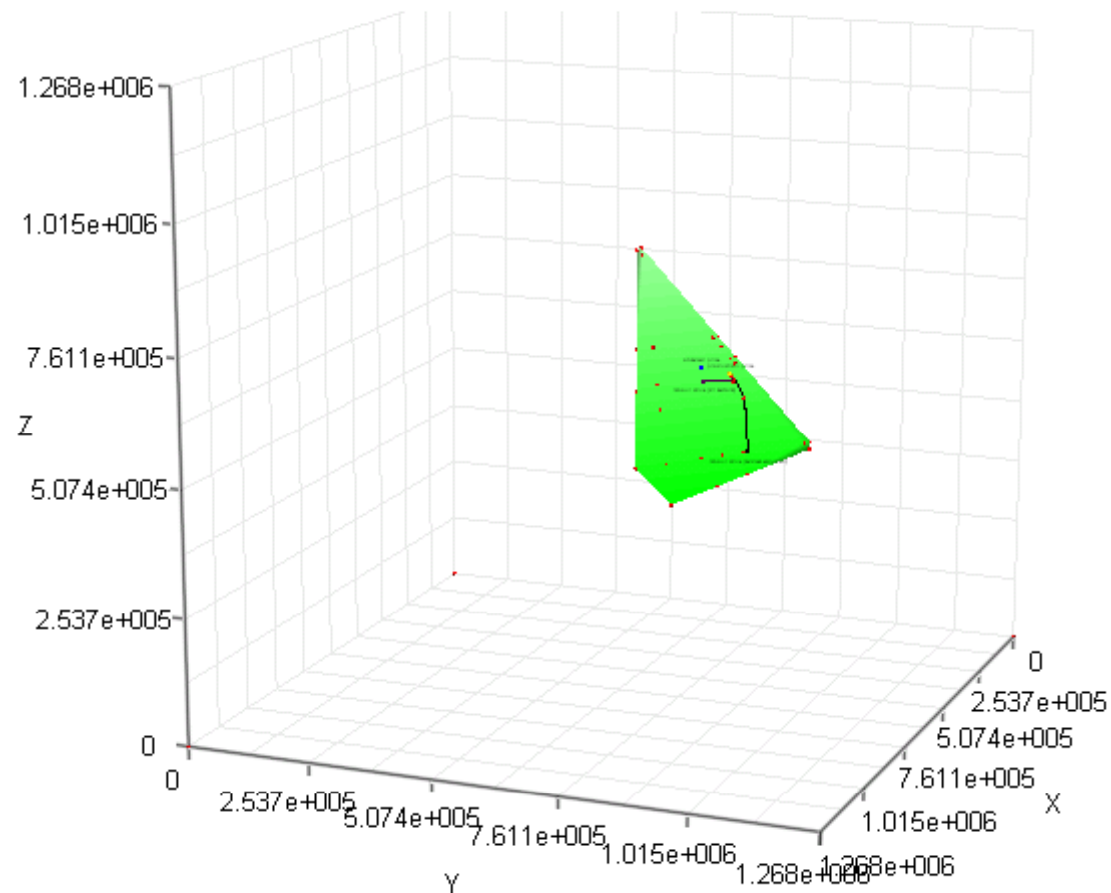
$$f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0 \quad f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0$$

hyperplane (Austria: 3 sectors)



$$f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0 \quad f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0$$

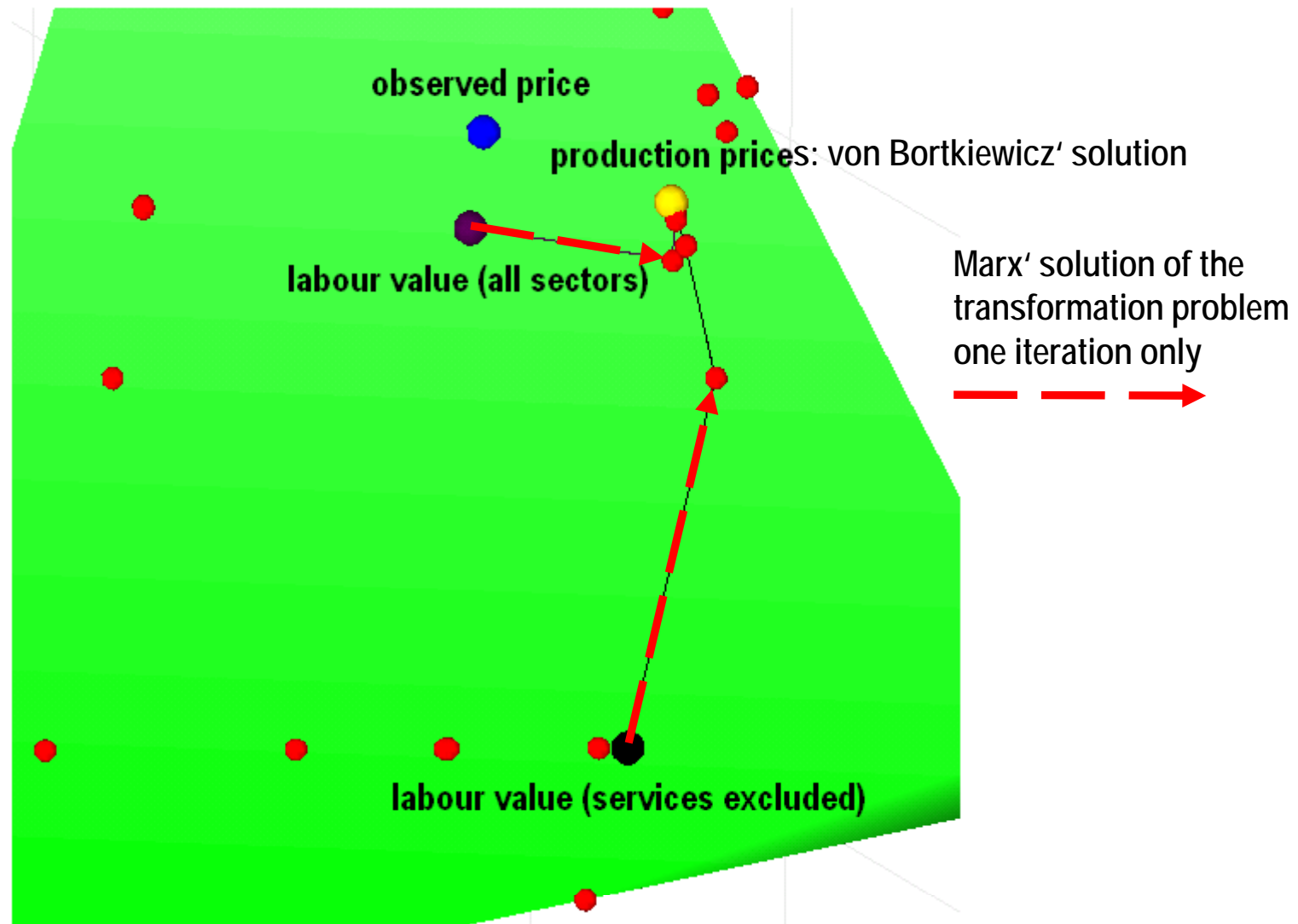
set of all feasible prices – corresponding to fixed surplus product, but variable distribution of profits: subset of hyperplane $px = \text{const}$ (Austria: 3 sectors)



$$f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0 \quad f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0$$

Prices and labor values

(hyperplane $px = \text{const}$)



$$f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0 \quad f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0$$

*Thank you
for your attention!*

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$$f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0 \quad f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \text{ for } x > 0$$